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External Expert's Opinion
on the Activities of the COLABS Project in Poland

1. Introductory remarks

Five "mathematical microworlds" addressed to the students aged 12 to 18 were designed and verified in practice by OEIiZK under the Colabs project. The microworlds allow and facilitate active learning of chosen areas of mathematics by students' own intellectual activity: their own explorations and trials. Each microworld is accompanied by materials and directions for the students as well as comments designed for the teacher. I will not dwell here on the mathematical matter of individual microworlds presenting only a brief outline. Instead I will concentrate on the educational value of the project in view of the current Polish educational system and its needs.

2. Mathematical microworlds

q Polygons

This microworld deals with the topics which appear both at primary school and at the further stages of the educational process. Its design allows it to be used with students of very different ages and levels of mathematical competence, beginning with forms 4-6.

Working in the microworld a student can construct various polygons and explore their properties. They can make geometrical experiments not only on the "synthetic" properties of the shapes but also on the lengths of segments, angle measures and areas of polygons. Based on these experiments they can construct their knowledge. In the process they become acquainted with important mental operations such as generalisation, specification or reasoning by analogy, and learn to use them.

The microworld is a tool for discovering and purposeful exploring the consequences of the changes made either in the parameters of a single shape of in a more complex shape structure. The student him/herself can define their area of interest and independently pose research problems relevant to them. The use of square grid as a tool for settling questions about the properties of the shapes under examination allows students to give reasons and develop argumentation.

Doing geometry, or maths in general, by experimenting has immense educational value, however, it calls for caution. This empirical approach should be reasonably balanced with building the right intuitions and geometrical correctness. It is therefore necessary to generate such situations, problems and questions, which unite both aspects. A large part of the activities offered by the microworld fulfils this requirement but some need modifying.

- **Vectors**

This microworld allows the students to carry out rich experiments on vectors and to build intuitions of the concept. It gives them an opportunity to get acquainted with different representations of vectors, further develop their skills connected with system (systems) of co-ordinates, and to gain a better understanding of a physical sense of vectors as well as the relations of the basic physical variables (velocity, force etc.) and the laws that govern them. Working with the microworld often becomes an involving play, for example at "space travel" and „hues composing".

Experiments and exercises presented in the microworld enrich students' experience, give them an opportunity to formulate and verify hypotheses, to make generalisations, in other words: to mathematical and physical creativity. Currently vectors appear at Polish middle and upper secondary school (gymnasium and lyceum). The level of complexity of the microworld allows it, in my opinion, to be used effectively at both these stages.

- **The laboratory of randomness**

I fully agree with the opinion expressed by the projects' authors that the topic of randomness is on one hand very important, not least because of its manifold practical applications, on the other quite difficult. This difficulty for Polish students arises partly from the way in which probability is taught at school. At the lower stages of education elements of it appear sporadically and if they do, it is a result of the involvement of the authors of one or two of more then twenty textbooks rather than common awareness of the significance of this field.

At the higher levels of education the teachers and the authors of curricula and textbooks often escape into mathematical formalism difficult for students. What such an approach lacks is awaking a wider scale "probabilistic awareness" in children, as well as early enough and consistent development of the right probabilistic intuitions.

The *Laboratory of Randomness* consists of a series of "probabilistic projects" showing different areas or aspects of randomness. Students' own activity takes them all the way from probabilistic experiments and experimental frequencies to theoretical models such as e.g. Engel's diagrams, which yield a better and more accurate description of the phenomena. Following this way is, I believe, necessary for a good understanding of the nature of randomness. The projects offered in the microworld go quite deep (sometimes very deep indeed) into randomness, touching important and

difficult probabilistic concepts (random variable, its distribution, mean, etc.). The authors apply different tools including Logo procedure definitions, which undoubtedly enriches the offer, although, as they admit themselves, does not necessarily make it any simpler.

- **Game algebra**

The role of games in mathematical education cannot be stressed enough. Games motivate, give an opportunity to carry out various important mathematical exercises in a pleasant setting, they can provide a realistic context for doing maths. Recent years have seen their introduction to Polish schools, although at present it is limited mostly to the earliest stages of education.

Games offered in this microworld are certainly involving, especially for people who like this kind of intellectual activity and its challenges. They allow the students to discover patterns, to purposefully develop local strategies, to see analogies and symmetries. Even the very stage of exploring those games is mathematically educational.

The idea of applying games in order to make students familiar with elements of algebra (sets of equations, Gauss procedure etc.) and to show them their usefulness, for example for constructing global winning strategies, is, in my opinion, original and interesting.

At the same time, however, the microworld poses a difficult challenge for a student. Even the activity of consistent search for a winning strategy is quite foreign to an average person. The tools used in the project, e.g. arithmetic modulo 2, create additional difficulties. Therefore I see a need to supplement the games with more examples in which the level of abstraction and difficulty rises more gently and gradually.

- **Visual Modelling**

The main goal of this microworld is learning the art of problem solving. In some of the problems presented a solution would not be possible at all without a computer. A computer also allows conscious and consistent application of the trial and error approach, or rather trial and correction, which is one of the most powerful and most natural tools of problem solving. While struggling with problems, which are very attractive visually, or even border on artistic, a student gets the opportunity to apply their knowledge and to deepen their understanding of mathematical ideas. They learn to translate what they see into mathematics, and discover the usefulness of e.g. trigonometry, which has been somewhat underestimated at Polish school. This kind of situation can motivate students far better than traditional activities of the teacher.

The problems presented are rich and mathematically varied: they involve circle geometry, trigonometry, various properties of shapes, the ideas of recurrence and randomness. They are also, in my opinion, quite compelling. As a result students merely apply the knowledge they already have rather than generating it during the problem solving; this, however, might have been the authors intention.

3. Educational value of the project seen from the Polish perspective

Education in Poland has been in the state of "permanent reform" for some time already, but the last couple of years have seen its further intensification. Major changes in the curriculum have been (and are still being) made and the very structure of the school system has been modified.
Basic curriculum was introduced as a "common core" of all the curricula used at schools. Among other tasks set before the school it lists so called key skills: creative problem solving, the ability to use information technology, co-operation, communication, After several years it can already be assessed that this area of planned changes has not been reflected on a larger scale in every day school practice. Unfortunately the administrative regulations were not followed by practical solutions.
We have forgotten about an old and frequently repeated truth that **true progress in education can be achieved not by changes in curricula or school types but only by perfecting the methods of work with children**. Meanwhile the successive waves of reform barely touched the teacher training system. It can also be assumed that, in spite of plans and efforts to do so, they did not change the educational philosophy of most Polish teachers. Which is, as practice shows, usually very traditional, based on behaviouristic principles, according to which the teacher is the "dispenser of wisdom" and the central figure of the learning process. The Polish school still prefers, regardless of the student's age, explicit instruction and concentrates on exacting the knowledge. Perhaps the reason of this state of affairs is the fact that a large part of the population including teachers is completely unaware that the learning process can and indeed should take a different course.

The Colabs project shows what a modern educational process (in which a student gets a chance to gain real knowledge) can look like and how modern technology can be used to change traditional methods of teaching and learning mathematics.
The microworlds present mathematics as an empirical (or quasi-empirical) science close to the vision of I. Lakatos, its only difference from e.g. physics being the nature of objects on which the experiments are conducted. In mathematics these are numbers, shapes, events, games...
With such an approach mathematical creativity comes naturally: experimenting is followed by hypotheses, their verification, examples and counterexamples. The students are intellectually active, in the zone of their proximal development, they co-create their knowledge in the process of communicating and co-operating with others. Not only the content is important but also strategies of acting in various situations. Motivation for studying is increased and the students become **ready to learn all their lives long**.

4. Final remarks

In my opinion the project shows exactly this educational style, which should be adopted by as many teachers as possible as soon as possible. It also advocates constructivism, which in Poland is only mentioned in the context of primary school if at all, as an effective approach also in work with much older students. Attempts to use microworlds at school look very promising.